PHYS 320 ANALYTICAL MECHANICS

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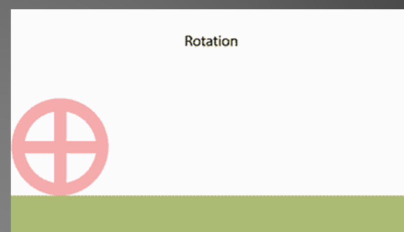
Fall 2108



TODAY

Vector review!

Newton's Laws

Vectors: triple vector products

- There are several (see text)
- Most likely to use:

volume of parallelepiped

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

Vector Derivatives

- Distributive: the derivative of a vector is the derivative of its components:

"r dot" means time derivative of r!

$$\dot{\vec{r}} \equiv \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\,\hat{x}) + \frac{d}{dt}(y\,\hat{y}) + \frac{d}{dt}(z\,\hat{z})$$

$$= \hat{x}\frac{d}{dt}(x) + x\frac{d}{dt}(\hat{x}) + \hat{y}\frac{d}{dt}(y) + y\frac{d}{dt}(\hat{y}) + \hat{z}\frac{d}{dt}(z) + z\frac{d}{dt}(\hat{z})$$

$$= \hat{x}\frac{d}{dt}(x) + \hat{y}\frac{d}{dt}(y) + +\hat{z}\frac{d}{dt}(z)$$

$$= \hat{x}\frac{dx}{dt} + \hat{y}\frac{dy}{dt} + +\hat{z}\frac{dz}{dt}$$

So, how do we write "speed" then?

Vector Derivatives

- Velocity and acceleration vectors:

$$\vec{v} \equiv \dot{\vec{r}} \equiv \frac{d\vec{r}}{dt} = \hat{x}\frac{dx}{dt} + \hat{y}\frac{dy}{dt} + +\hat{z}\frac{dz}{dt}$$

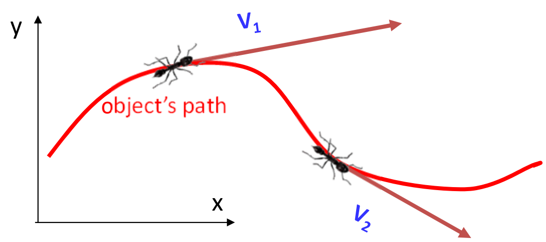
$$\vec{a} \equiv \dot{\vec{v}} \equiv \ddot{\vec{r}} \equiv \frac{d\vec{v}}{dt} = \hat{x}\frac{d}{dt}\left(\frac{dx}{dt}\right) + \hat{y}\frac{d}{dt}\left(\frac{dy}{dt}\right) + +\hat{z}\frac{d}{dt}\left(\frac{dz}{dt}\right)$$

$$= \hat{x}\frac{d^2x}{dt^2} + \hat{y}\frac{d^2y}{dt^2} + +\hat{z}\frac{d^2z}{dt^2}$$

$$= \hat{x}\frac{dv_x}{dt} + \hat{y}\frac{dv_y}{dt} + +\hat{z}\frac{dv_z}{dt}$$

Instantaneous Velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$



$\vec{v}$ is the rate of change of position and can be represented by a vector!

The velocity vector is always tangent to the path (trajectory) of motion in real space

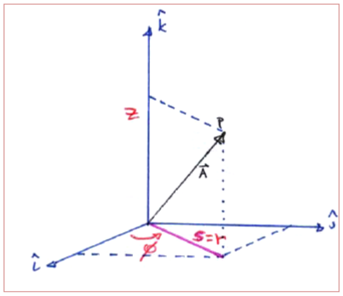
Cylindrical Coordinates

r = s = polar coordinate – radial (xy-plane)
ϕ = polar coordinate – angle (x-y-plane)
z = Cartesian z-coordinate

Unit vectors: $\hat{r} = \hat{s}, \hat{\phi}, \hat{k}$

$$x = r\cos\phi$$
$$y = r\sin\phi$$
$$z = z$$

$$\hat{r} = \hat{s} = \cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath}$$
$$\hat{\phi} = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}$$
$$\hat{k} = \hat{k}$$



$$s = \sqrt{x^2+y^2}$$
$$\phi = \tan^{-1}[y/x]$$
$$z = z$$

$$\hat{\imath} = \cos\phi\,\hat{r} - \sin\phi\,\hat{\phi}$$
$$\hat{\jmath} = \sin\phi\,\hat{r} + \cos\phi\,\hat{\phi}$$
$$\hat{k} = \hat{k}$$

$$d\vec{\ell} = dr\,\hat{r} + r\,d\phi\,\hat{\phi} + dz\,\hat{k}$$
$$d\tau = dV = r\,dr\,d\phi\,dz$$

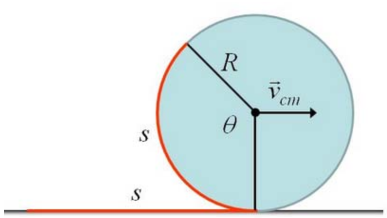
$d\vec{A}$ depends on surface!

DIV, GRAD, and CURL are no longer so simple!!

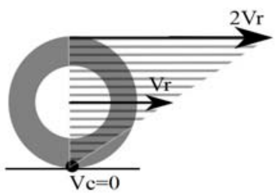
Rolling motion

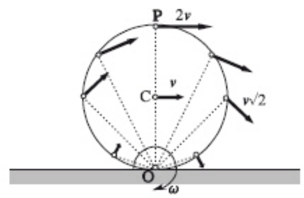
Radians (“unit” of angular measure) are defined as a *ratio* of two lengths.

$$\theta = s / r$$

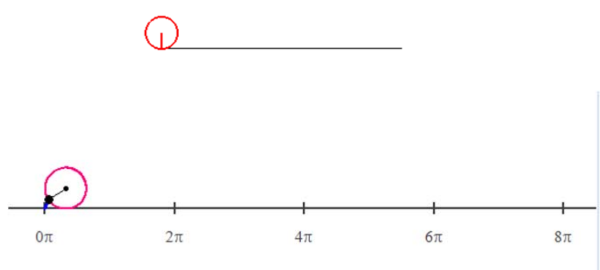


Rolling motion





Rolling motion



Path of point on the rim of the wheel is a cycloid.

Rolling motion

- How describe …
  - linear motion?

$$\vec{r}_1 = \hat{i}\, b\omega t + \hat{j}\, b$$

  - circular motion?

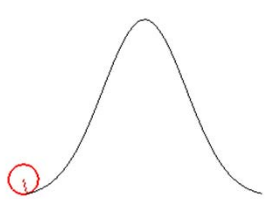
$$\vec{r}_2 = \hat{i}\, b \sin \omega t + \hat{j}\, b \cos \omega t$$

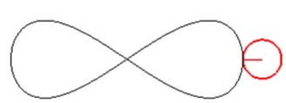
  - the combination of circular and linear motion: rolling motion?

$$\vec{r} = \vec{r}_1 + \vec{r}_2 = \hat{i}\, b(\omega t + \sin \omega t) + \hat{j}\, b(1 + \cos \omega t)$$

Rolling motion

For more general fun …





NEWTON'S LAWS

Valid only in INERTIAL reference frames!

- I Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impresses upon it.
- II The change of motion is proportional to the motive force impresses; and is made in the direction of the line in which that force is impressed.

Note: these two refer to a specific body (mass)